Shielded Biplanar Gradient Coil Design

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An application of the target field method to the design of shielded biplanar gradient coils for magnetic resonance imaging electromagnets is presented. Some specific cases are studied, and optimized geometries are proposed for the axial and transverse gradient coils that eliminate the third- and minimize the fifth-order terms in the magnetic field expansion. J. Magn. Reson. Imaging 1999;9:725–731. © 1999 Wiley-Liss, Inc.

Index terms: magnetic resonance imaging (MRI); planar target field; electromagnets; gradient fields

THE NECESSITY OF SHIELDING the time-dependent magnetic fields to avoid inducing eddy currents on the neighbor conductors is crucial in MRI, where strong magnetic gradients fields are switched in less than 1 msec. In superconducting MRI magnets, the induced eddy currents on the cryostat walls cause the quality of the gradient fields to deteriorate, especially in echoplanar imaging (EPI) sequences.

To solve this problem Turner (1–3) proposed a target field method that allows us to design accurate shielded magnetic fields in cylindrical geometry. This method is based on the use of a Fourier-Bessel expansion of the magnetic field generated by the current densities that flow on two cylindrical coaxial surfaces. An extended review of the method can be found in Turner (4).

The planar geometry for the gradient coils is also used in MRI, especially in permanent magnets and electromagnets. In these MRI systems the gradient set must be located very close to the magnetic poles to maximize the access without increasing the magnet gap. In electromagnet MRI systems, this causes strong eddy currents on the pole tips, requiring the use of a set of shielded gradient coils.

An extension of the target field method from cylindrical to planar geometries was proposed by Yoda (5) using, instead of the Fourier-Bessel expansion, the three-dimensional Fourier integral expansion of the magnetic field, generated by a planar current. Following Turner’s approach, this expansion was used to obtain a relationship between the Fourier components of the primary and shielding currents necessary to cancel the magnetic field on one side of the place. The real currents are then obtained by the inverse transform. A similar approach was used by Martens et al (6) and Lee et al (7) to design unshielded biplanar coils using a variational method to minimize inductance of the coil.

Here we follow Yoda’s (5) method for the design of a set of biplanar shielded gradient coils using Turner’s approach to optimize the current geometry. A high homogeneity gradient design is proposed that eliminates the third-order magnetic field components and minimizes the fifth-order components as well.

THEORY

As shown in Fig. 1, we consider a set of four planes perpendicular to the z-axis where the outer planes (called shield planes) are placed at $z = \pm d$ and the inner planes (called primary coil planes) are placed at $z = \pm a$ with $d > a$. The aim of this section is to obtain the relation between the current densities flowing in these planes that will produce a null magnetic field outside the outer planes and the desired magnetic field shape between the two inner planes.

Therefore the current density, $j(r)$, can be expressed in the following form:

$$j(r) = \left[ j_z^{a}(x, y) \hat{x} + j_z^{q}(x, y) \hat{y} \right] \delta(z - a) + \left[ j_z^{q}(x, y) \hat{x} + j_z^{a}(x, y) \hat{y} \right] \delta(z + a) + \left[ j_y^{q}(x, y) \hat{x} + j_y^{a}(x, y) \hat{y} \right] \delta(z - d) + \left[ j_y^{a}(x, y) \hat{x} + j_y^{q}(x, y) \hat{y} \right] \delta(z + d)$$

(1)

where the indices $a$ and $d$ denote the current density in the principal and shielding planes, respectively.

As was mentioned previously, the three-dimensional Fourier expansion of the $z$-component of the magnetic field, $B_z(r)$, generated by the current distribution on a plane at $z = 0$, is given in Yoda (5) as

$$B_z(r) = - \frac{\mu_0}{8\pi^2} \int \frac{k_z^2 + k_y^2}{k_y k_z} e^{ik_z(k_x x + k_y y)} d^3k$$

(2)
where $k = (k_x, k_y, k_z)$ is the reciprocal space vector and $j_x(k_x, k_y)$ is the Fourier transform of the $x$-component of the current density, which can be written in general form as

$$j_x^{(l)}(k_x, k_y) = \int \int j_x(x, y) e^{-ik_x x + ik_y y} \, dx \, dy$$

(3)

where $l = x, y$ and $l = \pm a, \pm d$. The continuity equation for the current density, used to derive Eq. [2], can be expressed in terms of the Fourier components of the current density as

$$k_x j_x(k_x, k_y) + k_y j_y(k_x, k_y) = 0.$$  

(4)

The $z$-component of the magnetic field, $B_z(r)$, must be evaluated in two different regions. a) outside the shield ($|z| > d$), and b) inside the primary planes ($|z| < a$). From Eqs. (1) and (2) and integrating over $k_x$, we find

$$B_x^2(r) = -\frac{\mu_0}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_x^2 + k_y^2}{k_y}$$

$$\left( j_x^{(a)}(k_x, k_y) e^{-ik_x x + ik_y y} \right)$$

$$+ j_x^{(d)}(k_x, k_y) e^{ik_x x - ik_y y}$$

$$+ \int \int j_x(x, y) e^{-ik_x x + ik_y y} \, dx \, dy$$

(5)

where $B_x^2$ refers to the outer region ($|z| > d$), and $B_x^2$ to the inner region ($|z| < a$).

Because of the inherent symmetry associated with the gradient fields used in MRI, it is convenient to consider separately symmetric ($f(z) = f(-z)$) and antisymmetric ($f(z) = -f(-z)$) current distribution. In fact, these two cases will be associated with generation of the transversal and longitudinal gradients, respectively, as will be done later.

For the present geometry using the previous notation we have $f_x^a = f_x$ and $f_x^d = -f_x$. Therefore we can write Eq. [5] as

$$B_x^2(r) = -\frac{\mu_0}{4\pi^2} \int (f_x^{(a)}(k_x, k_y) f_x^{(a)}(k_x, k_y)$$

$$+ f_x^{(d)}(k_x, k_y) f_x^{(d)}(k_x, k_y))$$

$$e^{-z\sqrt{k_x^2 + k_y^2}} \frac{k_x^2 + k_y^2}{k_y}$$

(6)

with

$$f_x^{(a)}(k_x, k_y) = \sinh (\alpha \sqrt{k_x^2 + k_y^2})$$

antisymmetric case

$$f_x^{(a)}(k_x, k_y) = \cosh (\alpha \sqrt{k_x^2 + k_y^2})$$

symmetric case

(7)

where $\alpha = a, d$ in Eq. [6].

For the shield to work, we require $B_x^2(r) = 0$ for $|z| > d$. Then from Eq. [6] the shield conditions are given by the equation

$$f_x^{(a)}(k_x, k_y) = \frac{f_x^{(a)}(k_x, k_y)}{f_x^{(a)}(k_x, k_y)}.$$  

(8)

Furthermore, it can be shown that the same arguments of Yoda (5) applied to the shielding condition in Eq. [8] result in the nulling of the transverse $x$- and $y$-components of the magnetic field for $|z| > d$.

The $B_z$ field inside the inner region, $|z| < a$, could be obtained from Eqs. [5] and [8] resulting in

$$B_z^2(r) = -\frac{\mu_0}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$$

$$\sinh [(d - a) \sqrt{k_x^2 + k_y^2}] f_x^{(a)}(k_x, k_y)$$

$$\frac{1}{k_y} \frac{k_x^2 + k_y^2}{k_y}$$

$$e^{-z\sqrt{k_x^2 + k_y^2}}$$

(9)

where, according to the geometries, $f_x^{(a)}(k_x, k_y)$ and $f_x^{(a)}(k_x, k_y)$ assume either of the forms given in Eq. [7] for $\alpha = d$ and $\alpha = z$, respectively.
APPLICATIONS

In this section we will show how the above theory can be used to design a set of planar self-shielded gradient coils. Optimized wire geometries are derived to cancel the third- and fifth-order terms in the expansion of the z-component of the magnetic field produced by those current configurations, to obtain a uniform set of gradient fields.

Longitudinal Gradient

The usual way to produce an axial gradient is to use an antisymmetric configuration consisting of a pair of identical circular planar coaxial coils with the primary current $I$, at $z = \pm a$, usually called a Maxwell pair. If the coil radius $R$ is $2a/\sqrt{3}$, the resulting $B_z$ produced by this current distribution is a field whose first nonlinear term is of the fifth order. The use of an extra pair of coils to shield the pole tips from the gradient fields modifies the situation, making it necessary to find a new optimal geometry by adjusting the $R/a$ ratio for primary coils.

To obtain the optimal configuration in this situation, we begin calculating the current distribution in the shielding planes. Following ref. 5, the Fourier transform of the primary current density can be written as

$$f^P(k_x, k_y) = 2\pi i R R_0 J_1(R_0 q) \sinh(qa) \frac{\sinh(aq)}{\sinh(qa)}$$

(10)

where $k_x + ik_y = qe^{ik_y}$ and $J_1(R_0 q)$ is the Bessel function of order 1.

The current shield density flowing in the shielding planes at $\pm d$ can be obtained from Eqs. [8] and [10] by using the Hankel transform of order 1

$$f^Q(r) = -IR \int_0^\infty q \sinh(qa) J_1(R_0 q) J_1(r q) dq$$

(11)

Eq. [11] was used to calculate the shield current density $j^Q$ using $R = 9.8$, $a = 10$ cm, and $d = 12$ and 15 cm. The dimensions of this configuration were chosen to fit a 26 or 30 cm electromagnet gap with 20 cm of magnet access.

For construction purposes the $j(r)$ must be taken discretely. We choose to approximate the continuous current by a set of wire loops of radius $r_n$ calculated following ref. 5 by making

$$(n - 0.5)I_s = \int_0^\infty j^Q(r) dr$$

(12)

Using the value $I_s = 0.08 I$ for the current flowing in the wires, the number of necessary loops is 10. The continuous line in Fig. 2 shows the calculated continuous $j^Q(r)$ for $d = 12$ cm, for which the position of the discrete wires is shown as open circles. The dashed line gives $j^Q(r)$ for $d = 15$ cm. The filled dot at $r = 9.8$ cm shows the radial position of the primary current. It is further seen from the two curves in Fig. 2 that increasing the separation between the shielding and primary planes results in a large reduction of the necessary shielding density, leading to a more efficient action of the shield coil although requiring a larger gap to maintain the same magnet access.

Figure 2 shows the expected shielding performance of these four coil assemblies. These curves are the $z$- and $r$-components of the magnetic field calculated at $z = 14$ cm using the Biot Savart law.

The dotted lines give the magnetic field due to the main coil while the continuous lines give the overall field, showing the shielding effect resulting from the discrete current distribution shown in Fig. 2. It can be seen that the maximum of the magnetic field intensity remaining 2 cm above the shielding plane at $d = 12$ cm is only 4% of the unshielded value. This performance can be improved, approaching a continuous distribution, by increasing the number of shielding wires. Note from the figure that while the shielding condition given by Eq. [8] was obtained, imposing the condition $B_z = 0$ for $|z| > d$ also results in nulling of the radial component in the same region as discussed earlier.

As we said before, when active shielding is present, the optimum primary coil radius does not equal the Maxwell radius. To obtain the optimum radius in this condition we begin by writing from Eqs. [4], [9], and [10]

$$B_z(r) = \frac{\mu_0 I}{2\pi} \int_0^\infty J_1(R q) \frac{\sinh(qa) e^{ik_y + k_y d}}{\sinh(qa)} dk_x dk_y$$

(13)
and we expand the position-dependent terms in Eq. [13] in a Taylor series. As the even order terms must be zero by symmetry, the third-order terms in the expansion are the first nonlinear contribution. To get the more uniform gradient field we need to cancel all the terms that vary as $z^3$, $zy^2$, $zx^2$, and $zxy$. This can be done by requiring that the factor $Q_4$ (5) common to these terms be zero; in our case we have

$$Q_4 = \int_0^\infty J_1(Rq) \frac{\sinh [(d - a)q]}{\sinh (dq)} - q^4 \, dq = 0. \tag{14}$$

The numerical solution of this equation is shown in Fig. 4a, where the optimal $R/a$ ratio is given as a function of $d/a$. As can be seen from this figure, when $d/a \rightarrow \infty$ the optimal coil radius approaches the Maxwell radius, but it falls rapidly as $d/a$ approximates 1, substantially reducing the high homogeneity region, as found in ref. 2 for the cylindrical geometry. To increase the uniformity of the longitudinal gradient, we need to eliminate at the same time the third- and fifth-order terms in the expansion of Eq. [13]. This can be done by requiring that $Q_4 = 0$ and that simultaneously

$$Q_6 = \int_0^\infty J_1(Rq) \frac{\sinh [(d - a)q]}{\sinh (dq)} - q^6 \, dq = 0 \tag{15}$$

To make this possible we have to introduce an extra degree of freedom. We do this by introducing additional coils at $z = \pm a$, and the corresponding shielding currents at $z = \pm d$. The new coaxial current distribution, in the $a$-plane, becomes

$$j^a(r) = I[n_1 \delta(r - R_1) + n_2 \delta(r - R_2)] \tag{16}$$

where $R_1$, $R_2$ are the radii of coils 1 and 2 respectively, and $n_1$, $n_2$ are the number of turns on each coil and $I$ is the series current flowing in them. We can now try to adjust the coil radius to exactly cancel the third-order while minimizing the fifth-order contributions. Thus to obtain the best possible configuration we looked for a

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**Figure 3.** Magnetic field plots of the axial coil as a function of the radial distance from the axis $r$, showing the shield performance. a: z-component. b: Radial component.

**Figure 4.** a: Optimal radius for the biplanar shielded Maxwell pairs. The dashed line corresponds to the unshielded case. b: Plots of the normalized longitudinal gradient field as a function of $z/a$ for the four configurations discussed in the text. I, dotted-dashed line; II, dotted line; III, dashed line; IV, solid line.
solution of the following system
\begin{align}
Q_4(R_1) + \eta Q_4(R_2) &= 0 \\
Q_6(R_1) + \eta Q_6(R_2) &= 0
\end{align}
(17)

where \( \eta = n_2 R_2 / n_1 R_1 \), and Eq. [17] implies the cancelation of the cubic term.

The solution of the above set of equations involving the integral of the transcendental function requires the use of a numerical method. For this purpose we developed a computational algorithm that reproduced Turner and Bowley’s (2) results for the cylindrical geometry with a coil ratio equal to 9/7.

In our case the best solution was found for \( \eta = 0.08 \), resulting in \( R_1 / a = 1.40 \), essentially equal to the Maxwell radius, and \( R_2 / a = 0.57 \).

Figure 4b shows the normalized gradient field as a function of \( z / a \) for four coil configurations: I) the standard Maxwell pair; II) the same Maxwell pair shielded by the continuous current distribution given by Eq. [11] shown in Fig. 2; III) the same configuration using the \( R / a \) ratio that eliminates the third-order power according to Eq. [14]: the optimal configuration for \( d / a = 1.2 \) results in \( R / a = 0.98 \), as shown in Fig. 4a; and IV) the double ring design discussed above that minimizes the fifth-order terms.

As can be seen from Fig. 4, while the optimal \( R / a \) ratio practically recovers the field of view (FOV) loss resulting from the shielding, only the minimization of the fifth-order term results in an FOV larger than that of the unshielded Maxwell pair. Similar calculations were done for the radial field dependence for the same four cases.

The usable FOV (defined as the cylindrical region where the gradient field deviates from the iso-center value less than 5%) was calculated for all cases, and the radius \( r_{FOV} \) and length \( z_{FOV} \) are given in Table 1 in units of \( a \). The last column in Table 1 gives the efficiency for the different gradient systems relative to the basic Maxwell pair.

The last row shows that the proposed configuration results in a substantial improvement over the other shielded designs at the price of a tolerable loss in efficiency.

**Transverse Gradient**

To produce a transversal \( y \)-gradient we will consider as a primary current density two infinitely straight wires parallel to the \( x \)-axis on each of the planes at \( z = \pm a \). Thus the \( x \)-component of this symmetric current density can be written as
\[ j_x(x, y) = I [\delta(y + l) + \delta(y - l)] \]
(19)

where \( 2l \) is the distance between the wires.

The Fourier transform of this primary current distribution is
\[ \hat{j}_x(k_x, k_y) = 4 \pi I \delta(k_y) \cos k_y l. \]
(20)

From this, using the shielding condition of Eq. [8], the Fourier transform of the shield current distribution can be written as
\[ \hat{j}_s(k_x, k_y) = -4 \pi I \delta(k_y) \cos (k_y l) \frac{\cosh (\alpha \sqrt{k_x^2 + k_y^2})}{\cosh (d \sqrt{k_x^2 + k_y^2})} \]
(21)

which leads to
\[ j_s(y) = -\frac{I}{\pi} \int_{-\infty}^{\infty} \cos (k_y l) \cos (k_y y) \frac{\cosh (k_y d)}{\cosh (k_y d)} \, dk_y \]
(22)

This symmetric function of \( y \) is plotted in Fig. 5 for \( y > 0 \), \( l = 4.14 \) cm and the same \( a \) and \( d \) as before. An appropriate approximation to this current density using 20 straight wires carrying a current of 0.05\( I_1 \) is also shown in the figure.

![Figure 5](image-url)

**Table 1**

<table>
<thead>
<tr>
<th>Coil</th>
<th>( r_{FOV} / a )</th>
<th>( z_{FOV} / a )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.54</td>
<td>0.44</td>
<td>1.00</td>
</tr>
<tr>
<td>II</td>
<td>0.32</td>
<td>0.20</td>
<td>0.43</td>
</tr>
<tr>
<td>III</td>
<td>0.44</td>
<td>0.33</td>
<td>0.56</td>
</tr>
<tr>
<td>IV</td>
<td>0.93</td>
<td>0.58</td>
<td>0.34</td>
</tr>
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</table>

*Relative fields of view and efficiencies for the axial gradient coils arrangement are discussed in the text.*
Figure 6 shows the calculated $z$- and $y$-component of the magnetic field generated by the discrete current distribution shown in Fig. 5, together with the corresponding unshielded field. The coil length $L$ was limited to 40 cm for this calculation. This truncation did not have important consequences on the shield performance. As can be seen from Fig. 6, a maximum 6% of the magnetic field intensity survives 2 cm above the shielding plane at $z = 12$ cm.

It is interesting to note that, unlike the axial gradient case, generation of the transversal gradient requires a current distribution that has even symmetry with respect to $z$. Because of this even symmetry and since the shielding current in Eq. [22] drops rapidly to zero with $y$, it is possible to prove, using Ampere’s law, that the integral of the shielding current density, $j_0^0(y)$, equals the primary current $I$. Therefore it is possible to use the shielding plane as a return path for the primary current.

As in the longitudinal gradient case discussed above, this shield current density must be taken into account to choose the optimal parameter $l/a$ to get third-order cancelation of the field. The $z$-component of the magnetic field derived from Eqs. [9] and [20] may be expressed by

$$B_z(r) = -\frac{\mu_0 I}{\pi} \int_0^\infty \frac{\sinh \left[ (d - a)|k_y| \right]}{\cosh (k_yd)} \cdot \cosh (k_yd) \cdot \cos (k_yz) \cdot e^{ik_yy} \, dk_y$$

and to remove the odd $n$-order terms in $y$ and $z$, we require that

$$Q_n = \int_0^\infty \frac{\sinh \left[ (d - a)|k_y| \right]}{\cosh (k_yd)} \cdot \cos (k_yz) \cdot e^{ik_yy} \, dk_y = 0.$$  (24)

Solving Eq. [24] numerically we can obtain the optimal ratio $l/a$ as a function of $d/a$. The results are plotted in Fig. 7a, where the horizontal dotted line is the

Figure 6. Magnetic field plots of the transversal coil as a function $y$, showing shield performance. **a:** $z$-component. **b:** $y$-component.

Figure 7. **a:** Optimal radius for the biplanar shielded Golay arrangement. The dashed line corresponds to the unshielded case. **b:** Plots of the normalized transversal gradient field as a function of $z/a$ for the four configurations discussed in the text. 1, dotted-dashed line; 2, dotted line; 3, dashed line; 4, solid line.
asymptotic value \( \tan \left( \frac{\pi}{8} \right) \) for \( d/a \to \infty \), as in ref. 8. As in the axial case, the gradient uniformity can be improved by using additional wires, adding to each of the principal planes an extra pair of wires separated by \( 2l' \) and carrying a current \( I' \). Following the same steps as before, we write

\[
J_{x}(x, y) = I[\delta(y + l) + \delta(y - l)] + I'[\delta(y + l') + \delta(y - l')]
\]

and solve the following system

\[
Q_3(l) + 3Q_3(l') = 0
\]

and

\[
Q_3(l) + 3Q_3(l') = 0
\]

for \( \zeta = I'/I \). Figure 7b shows the normalized gradient field as a function of \( z \) for the four coil configurations: 1) the four straight wire current distribution unshielded with \( l/a = 0.415 \); 2) the same shielded with \( d/a = 1.2 \); 3) the same as 2 using the optimal \( l/a = 0.370 \) obtained from Fig. 7a; and 4) the eight-wire shielded arrangement with \( \zeta = 0.73 \), \( l/a = 0.8 \) and \( I'/a = 0.23 \). In all the cases the coil length \( L \) was set to \( 4a \).

<table>
<thead>
<tr>
<th>Coil</th>
<th>( x_{FOV}/a )</th>
<th>( y_{FOV}/a )</th>
<th>( z_{FOV}/a )</th>
<th>( \epsilon )</th>
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<td>0.44</td>
<td>0.33</td>
</tr>
</tbody>
</table>

*Relative fields of view and efficiencies for the transversal gradient coils arrangement are discussed in the text.

Table 2 shows the usable prismatic FOV for each of the above cases and the corresponding efficiency.

**CONCLUSIONS**

Yoda’s extension of Turner’s target field method was used to design a set of biplanar shielded gradient coils suitable for use in an MRI electromagnet. The geometry and shield current distribution were optimized for a basic set of primary gradient coils composed of two-loop Maxwell and four straight wire Golay arrangements. A modified geometry is proposed that minimizes the fifth-order terms in the field expansion, resulting in an extended FOV. The performances of these two systems are compared with the unshielded simple set in terms of FOV and current efficiency. The methods for obtaining the optimal parameter for the proposed geometry are given, and the design of the shielding coils is discussed. It is pointed out that the shielding coils can serve as the return path for the primary current in transverse coils.

**REFERENCES**